Lecture Notes on Law and Economics: Theory

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These lecture notes are an attempt to synthesize the various source materials for this course under a single framework with unified notation.

1 Introduction to Legal Incentives

This section presents basic models of the key problems solved by property law, contract law, tort law, and criminal law.

1.1 Property Law

Consider a landowner A and his neighbor, B. A chooses a binary action, $x \in \{0, 1\}$. The classic example is that A is a rancher and B is a farmer. $x = 1$ means he builds a fence, which would protect B's crops from the cattle trodding over them. $x = 0$ means the fence is not built. A and B begin with endowments ω_A and ω_B. A monetary transfer between the parties is represented by w. The cost of x to A is parametrized by a and the benefit of x to B is parametrized by b. The payoff functions for A and B are

$$u_A = \omega_A + w - ax$$

and

$$u_B = \omega_B - w - b(1 - x).$$

In the classic story where the rancher has to build a fence, we think of a as the cost of building the fence, while b is the damage done to the farmer's crops if the fence is not built.

Now consider two different legal regimes. In Regime 1 ("Rancher's Rights"), A is not responsible for any damages caused by his cattle. In Regime 2 ("Farmer's Rights"), A has to pay for any damages caused by his cattle.

In Regime 1, the payoffs for A and B for $x \in \{0, 1\}$ are:

$$u_A(x=0, w=0) = \omega_A$$

$$u_A(x=1, w=0) = \omega_A - a$$

$$u_B(x=0, w=0) = \omega_B - b$$

$$u_B(x=1, w=0) = \omega_B$$

In the absence of an agreement, A chooses $x = 0$ because $\omega_A \geq \omega_A - a$.

In Regime 2, the payoffs are

$$u_A(x=0, w=0) = \omega_A - b$$

$$u_A(x=1, w=0) = \omega_A - a$$

$$u_B(x=0, w=0) = \omega_B$$

$$u_B(x=1, w=0) = \omega_B$$

reflecting that A is going to pay B's damages when $x = 0$. In this case, A chooses $x = 1$ if $a < b$ and $x = 0$ if $a > b$. Note that Regime 2 results in the socially optimal outcome given the structure of the problem, since A internalizes the costs of the externality.

Now return to Regime 1, but allow bargaining between A and B. The neighbors will write a contract specifying a fence choice $\tilde{x}$ and a transfer w. In particular, B can pay A to build the fence. In order to persuade A to build the fence, B must offer $w \geq a$. But B stands to gain only if $w \leq b$. The range of w for which both A and B benefit from the fence is $w \in [a, b]$, but this interval exists only when $a \leq b$. Therefore an agreement to build the fence will only occur when the benefit of the fence b is greater than or equal to the cost of the fence a. If $a < b$, not building the fence is not an equilibrium because B would improve his payoff by offering $w \geq a$ and having the fence built. If $a > b$, building the fence is not an equilibrium because B would be paying $w > b$, in which case he would get a better payoff by abandoning the deal and allowing his crops to be trampled. If $a = b$, A and B are indifferent between having or not having a fence (and therefore an agreement). This coincides with choosing x to optimize the social welfare function

$$W = u_A + u_B = \omega_A + \omega_B + (b - a - 1)x.$$

Now return to Regime 2 allowing for an agreement $(\tilde{x}, w)$. In this case A must pay B if the crops are trampled. As with the case where no bargaining was allowed, A will build the fence when $b > a$ to avoid paying damages. When $a < b$, he won't build a fence and will go ahead and pay the damages. Therefore, the outcome is the same as under Regime 1. This again results in the social optimal outcome on x.

In the absence of transaction costs, default property rights don't matter. This result is known as the Coase Theorem.

1.2 Contract Law

Consider a seller A and a buyer B. A promises to sell to B a good with quality $x \geq 0$ at price w, but B will observe the quality only after paying w. The cost of quality for A is ax, while the benefit of quality to B is $b(x)$, where we assume $b(0) = 0$, $b'(\cdot) > 0$, and $b''(\cdot) < 0$. The payoff from the transaction for the seller is

$$u_A = w - ax$$

and the buyer's utility is given by

$$u_B = b(x) - w$$

We assume the outside options for each agent are such that payoffs are zero without a contract. So the transaction is mutually beneficial for (x, w) such that $w \geq ax$ and $b(x) \geq w$, which can be written as $ax \leq w \leq b(x)$.

The parties have written a contract, and B has paid w. What will A do? In the absence of any legal incentives, A chooses $x = 0$ and earns payoff w; B receives the good and observes quality $x = 0$ and is left with payoff $-w$. Looking ahead from the beginning of the relationship, B will see this coming and refuse to enter the transaction. Both parties get a zero payoff.

Now take the case of enforceable contracts. After receiving the good, B observes quality x and see if it corresponds to the contracted level $\tilde{x}$. If $x < \tilde{x}$, B can file a claim for breach of contract. With the standard rule of expectations damages, A will be forced to compensate B for any difference between his contracted payoff level $b(\tilde{x}) - w$ and the actual payoff $b(x) - w$. That is, A will pay damages z such that

$$b(\tilde{x}) - w + z = b(x) - w$$

which means

$$z(x) = b(\tilde{x}) - b(x).$$

Under this system of legal incentives, the seller's payoff as a function of x when $x \leq \tilde{x}$ is

$$\begin{aligned} u_A &= w - ax - z \\ &= w - ax + b(x) - b(\tilde{x}). \end{aligned}$$

$A's$ choice of x maximizes this function, which gives

$$b'(x) = a$$

Which is the same quality level as if he just followed the contract. Define x^* as the quality level satisfying this condition. This level is socially efficient. At the social optimum, the marginal benefit of quality equals the marginal cost, which is satisfied under contract enforcement with expectations damages.

The equilbrium contracts are the set of contracts $(\tilde{x}, \tilde{w})$ where $\tilde{x} = x^*$ and w satisfies the participation constraints

$$w - ax^* \geq 0$$

and

$$b(x^*) - w \geq 0$$

which means

$$ax^* \leq w \leq b(x^*).$$

This means that a mutually beneficial contract exists, and the transaction will occur, when

$$ax^* \leq b(x^*)$$

This is better than the situation without legal enforcement of contracts, where the transaction can never occur in equilibrium.

Example. Let $b(x) = \frac{1}{\beta}x^\beta$ with $\beta \in (0, 1)$. What is the contracted quality level $\tilde{x}$ without contract enforcement? With contract enforcement? What are the damages paid for quality $x < \tilde{x}$? What is the range of w for which a Pareto optimal transaction exists?

Solution. Without contract enforcement, the buyer knows that the seller will renege on any contracted level $\tilde{x} > 0$ and choose $x = 0$. So no contract occurs. With contract enforcement, the buyer chooses $\tilde{x} = x^*$, the socially optimal level where $b'(x) = a$. Here, $b'(x) = x^{\beta-1}$. So we have

$$
\begin{aligned}
x^{\beta-1} &= a \\
x^* &= a^{\frac{1}{\beta-1}}.
\end{aligned}
$$

How do we know he chooses this level? It is the only equilibrium with free negotiation. If the buyer proposed $\tilde{x} < x^*$, he could increase the size of the surplus by choosing $\tilde{x}'$ such that $\tilde{x} < \tilde{x}' \leq x^*$, which for constant w would increase the payoff to the buyer without decreasing the payoff to the seller. The same is true for $\tilde{x} > x^*$.

Under expectations damages, the seller will pay $z(x) = b(\tilde{x}) - b(x)$ for choosing $x < \tilde{x}$. For the functional for we have

$$
\begin{aligned}
z(x) &= b(\tilde{x}) - b(x) \\
&= \frac{1}{\beta}\tilde{x}^{\beta} - \frac{1}{\beta}x^{\beta}
\end{aligned}
$$

for $\tilde{x} = x^* = a^{\frac{1}{\beta-1}}$, this is

$$
\begin{aligned}
z(x) &= \frac{1}{\beta}(a^{\frac{1}{\beta-1}})^{\beta} - \frac{1}{\beta}x^{\beta} \\
&= \frac{a^{\frac{\beta}{\beta-1}} - x^{\beta}}{\beta}
\end{aligned}
$$

The range of w for which a mutually beneficial transaction exists is

$$
\begin{aligned}
w &\in [ax^*, b(x^*)] \\
&\in [a^1 a^{\frac{1}{\beta-1}}, \frac{1}{\beta}(a^{\frac{1}{\beta-1}})^{\beta}] \\
w^* &\in [a^{\frac{\beta}{\beta-1}}, \frac{a^{\frac{\beta}{\beta-1}}}{\beta}]
\end{aligned}
$$

Because $\beta < 1$, the second term in the interval is greater than the first term. A more concave utility function$b(x)$ means a larger contracting surplus.

1.3 Tort Law

Consider an injurer A and a victim B. A is performing some activity, such as driving, that poses some risk of harm b to B. A chooses some level of precaution $x \geq 0$ that has a cost a but reduces the probability that the harm to B occurs. The payoff function for A is

$$u_A = \omega_A - ax$$

where ω_A is an endowment. The harm occurs with probability $1 - p(x)$, where $p(x)$ is strictly increasing and strictly concave in x. If the harm does not occur B receives

$$u_B = \omega_B$$

and if the harm does occur he receives

$$u_B = \omega_B - b$$

where ω_B is again an endowment. The expected payoff to B is

$$\mathbb{E}(u_B) = \omega_B - (1 - p(x))b$$

Without tort law, A gives no concern to the externality imposed on B – he chooses $x = 0$ and earns his endowment ω_A. B's expected utility is $\omega_B - (1 - p(0))b$. Under the traditional compensation rule in tort, A just has to compensate B for any harm inccurred. Under tort law, B's utility does not depend on whether the harm occurs because he is perfectly compensated when it does occur:

$$u_B = \omega_B$$

The injurer now bears the risk. Without the harm, he receives

$$u_A = \omega_A - ax$$

and with the harm he receives

$$u_A = \omega_A - ax - b$$

The expected utility is

$$\begin{aligned}\mathbb{E}(u_A) &= p(x)(\omega_A - ax) + (1 - p(x))(\omega_A - ax - b) \\ &= \omega_A - ax - (1 - p(x))b\end{aligned}$$

Choosing x to maximize expected utility gives

$$a = p'(x)b$$

let x^* be the precaution level that satisfies this condition. This is the socially optimal precaution level.

Note that under tort law incentives, A will not engage in the activity unless there is a net social benefit to the activity.

Example. Let $p(x) = 1 - \frac{\rho}{x}$. What is the precaution level and resulting utilities without tort law? With tort law?

Solution. Without tort law, $x = 0$. $u_A = \omega_A$ and

$$\begin{aligned}
\mathbb{E}(u_B) &= \omega_B - (1 - p(x))b \\
&= \omega_B - (1 - (1 - \frac{\rho}{x}))b \\
&= \omega_B - \frac{\rho}{x}b \\
\mathbb{E}(u_B|x = 0) &= \omega_B - \frac{\rho}{0}b \\
&\rightarrow -\infty
\end{aligned}$$

Fortunately for B, tort law relieves him of risk: $u_B = \omega_B$. The injurer now has

$$\begin{aligned}
\mathbb{E}(u_A|x) &= \omega_A - ax - (1 - p(x))b \\
&= \omega_A - ax - (1 - (1 - \frac{\rho}{x}))b \\
&= \omega_A - ax - \frac{\rho b}{x}
\end{aligned}$$

Take the derivative for x:

$$\begin{aligned}
0 &= -a + \frac{\rho b}{x^2} \\
ax^2 &= \rho b \\
x^* &= \sqrt{\frac{\rho b}{a}}
\end{aligned}$$

Which gives expected utility

$$\begin{aligned}
\mathbb{E}(u_A|x = \sqrt{\frac{\rho b}{a}}) &= \omega_A - a\sqrt{\frac{\rho b}{a}} - \frac{\rho b}{\sqrt{\frac{\rho b}{a}}} \\
&= \omega_A - 2\sqrt{\rho ab}
\end{aligned}$$

1.4 Criminal Law

Consider a criminal A who chooses a level of criminal activity with severity $x \geq 0$. The activity accrues private benefits ax to the criminal but imposes a negative externality on the rest of the society – the classic example is tax evasion, where x is how much tax to evade in dollars. The criminal is caught with some probability $p(x, y)$, where y is the amount of tax money invested by the government in police and other law enforcement efforts. The assumptions are $p_x > 0$, $p_y > 0$, $p_{xx} < 0$,$p_{yy} < 0$, $p_{xy} > 0$ (where subscripts denote partial derivatives). The criminal law specifies a punishment $b(x)$ that the criminal bears in the event of detection, were $b_x > 0$.

When he is not detected, the criminal's utility function is

$$u_A = ax.$$

When he is detected and punished, his utility is

$$u_A = ax - b(x).$$

The expected utility is

$$\begin{aligned}\mathbb{E}(u_A) &= (1 - p(x, y))ax + p(x, y)(ax - b(x) \\ &= ax - p(x, y)b(x)\end{aligned}$$

The criminal maximizes this equation – taking y as given, the optimality condition is

$$a = p_x(\cdot)b(\cdot) + p(\cdot)b_x(\cdot)$$

A higher marginal benefit to crime a will increase the chosen crime level, while an increase in the punishment function $b(\cdot)$ will results in less crime.

Let $x^*(y)$ be the criminal's optimizing crime choice given the enforcement expenditure y. Let's say that society's welfare function does not depend on whether the criminal is caught for his crimes – compensation is impossible:

$$W = \omega - y - x^*(y)$$

In that case crime enforcement expenditure have no social benefit beyong reducing the crime level x. In particular, the social optimum requires

$$1 = -x^*_y(y)$$

That is, the marginal cost of tax expenditures on enforcement (assumed to equal one in this case) equals the marginal reduction in crime.

Example. Let $b(x) = x$ and Let $p(x, y) = x^{1/2}y$. Solve for the crime level as a function of y, and then solve for the optimal enforcement expenditure y.

Solution. We have $b_x = 1$ and

$$p_x = \frac{\partial p(\cdot)}{\partial x} = \frac{1}{2}x^{-\frac{1}{2}}y$$

Using the criminal's optimality condition

$$a = p_x(\cdot)b(\cdot) + p(\cdot)b_x(\cdot)$$

gives

$$\begin{aligned}
a &= [\frac{1}{2}x^{-\frac{1}{2}}y]x + x^{\frac{1}{2}}y \\
a &= \frac{3y}{2}x^{\frac{1}{2}} \\
\frac{2a}{3y} &= x^{\frac{1}{2}} \\
x^*(y) &= \frac{4a^2}{9y^2}
\end{aligned}$$

Which is the criminal's best response function to the government enforcement expenditure, decreasing in y, as desired. To get the the optimal government expenditure, take the social welfare function

$$\begin{aligned}
W(y) &= \omega - y - x^*(y) \\
&= \omega - y - \frac{4a^2}{9y^2}
\end{aligned}$$

and take the derivative for y:

$$\begin{aligned}
\frac{\partial W}{\partial y} &= -1 + \frac{8a^2}{9y^3} \\
1 &= \frac{8a^2}{9y^3} \\
y^3 &= \frac{8}{9}a^2 \\
y^* &= (\frac{8}{9})^{\frac{1}{3}}a^{\frac{2}{3}}
\end{aligned}$$

gives the government's optimal crime enforcement expenditure.

Now plug this back into the criminal's best response function:

$$\begin{aligned}
x^*(y) &= \frac{4a^2}{9}y^{-2} \\
&= \frac{4}{9}a^2((\frac{8}{9})^{\frac{1}{3}}a^{\frac{2}{3}})^{-2} \\
&= \frac{4}{9}a^2 \cdot (\frac{8}{9})^{-\frac{2}{3}}a^{-\frac{4}{3}} \\
x^* &= 3^{-\frac{2}{3}}a^{\frac{2}{3}}
\end{aligned}$$

The probability of detection $x^{1/2}y$ is

$$\begin{aligned}
p(x,y) &= x^{\frac{1}{2}}y \\
&= (3^{-\frac{2}{3}}a^{\frac{2}{3}})^{\frac{1}{2}}((\frac{8}{9})^{\frac{1}{3}}a^{\frac{2}{3}}) \\
&= 3^{-\frac{1}{3}}a^{\frac{1}{3}}(\frac{8}{9})^{\frac{1}{3}}a^{\frac{2}{3}} \\
p(x^*,y^*) &= \frac{2}{3}a
\end{aligned}$$

Expected utility for the criminal is

$$\begin{aligned}
\mathbb{E}(u_A) &= ax - p(x,y)b(x) \\
&= a(3^{-\frac{2}{3}}a^{\frac{2}{3}}) - \frac{2}{3}a(3^{-\frac{2}{3}}a^{\frac{2}{3}}) \\
&= 3^{-\frac{2}{3}}a^{\frac{5}{3}} - \frac{2\sqrt[3]{3}}{9}a^{\frac{5}{3}} \\
&= \frac{\sqrt[3]{3}}{9}a^{\frac{5}{3}}
\end{aligned}$$

And finally, social welfare for society is given by

$$\begin{aligned}
W^* &= \omega - y^* - x^* \\
&= \omega - (\frac{8}{9})^{\frac{1}{3}}a^{\frac{2}{3}} - 3^{-\frac{2}{3}}a^{\frac{2}{3}} \\
&= \omega - 3^{\frac{1}{3}}a^{\frac{2}{3}}
\end{aligned}$$

Note how these items change with the benefit from crime, a.

2 Rules Versus Taxes

Most textbook discussions of law and economics (including Cooter and Ulen) ignore an important problem with the use of legal rules. In particular: Why use legal rules rather than taxes? The standard models in public economics have figured out how to deal with externalities by implementing an optimal Pigouvian tax. This section discusses reasons to use rules rather than taxes, an analysis motivated by O'Flaherty (2005, ch. 8 sec. 2).

2.1 Pigouvian Taxes versus Legal Rules

Now some excerpts from O'Flaherty, 2005, Chapter 8:

- Neoclassical economists generally like prices a lot more than rules (by rules here I mean rules about quantities—how much you can spit on the street—not rules about prices). The reasoning should be familiar: if the government or the firm has enough information to set a price so it reflects the true (marginal) cost of an activity, then the only instances of an activity that will occur are those in which benefits exceed costs (to everyone), and these are precisely the instances that should occur if no further potential Pareto im- provements are to be possible. Rules, in contrast, appear to neoclassical economists to be blunt instruments: they don't differentiate well between those instances of an activity that should be done and those that should not.

- The neoclassical case against rules, however, is almost surely overstated. It compares ideal prices—the prices you could set only with excellent information—with actual rules—rules set in a context of poor information. In fact, if a government (or a firm) has the superb information it needs to set precisely the right prices, then it can set rules that work just as well as prices: it can permit those instances in which benefits exceed costs, and prohibit the others. When information is very good, rules work just as well as prices. The real question, then, is whether rules work better than prices when the information the government has is poor. The answer here is the usual, "It depends."

- Sometimes collecting money is simply infeasible under current technology, as at a traffic intersection or when a building is on fire. Other times the external harm of an act is so great that the optimal number of times for the act to be committed is clearly zero. Arson and murder (except in self-defense) are examples of this case.

- Rules work better than prices when the marginal external cost of what the public is doing is very sensitive to the quantity of what the public is doing, and when the marginal private benefit is not very sensitive to the quantity. Prices work better than rules in the opposite cases.

- For instance, a traffic intersection should be governed by rules because the marginal external cost of one car going through—about zero—is a lot less than the marginal external cost of two cars going through at the same time—a collision—while the marginal private benefit of each passage does not vary as much.

- Consider a small cruise ship with enough lifeboats for 100 people. The ship has 20 crew members... Thus each passenger beyond the eightieth endangers an additional crew member, while the first 80 passengers pose no such danger. The marginal social cost of carrying passengers beyond a total of 80 is much greater than the marginal social cost of carrying 80 passengers. If you're running the cruise ship ... you'll set a rule: no more than 80 passengers. You know the quantity of people beyond which marginal passengers harm the crew, but you don't know for sure how many passengers you'll have at a given price. If you set a price without a quantity rule and more than 80 passengers book passage, the consequences could be tragic. So when you don't know demand but you do know a quantity at which marginal social cost changes (80), it makes sense to use a rule rather than a price.

- In contrast, consider a copy machine in a public library. The library must replace the machine's ink, toner, and paper when they run out. Since all of these items are for sale in regular stores, the cost of a copy to the library—the marginal social cost—is pretty much the same no matter how many copies have been made in a given day. There's no magic number of copies after which the marginal cost suddenly jumps up. So setting a rule such as "Only 80 copies a day" would be a bad idea for the library. If patrons happened to want to make 200 copies some day and were willing to pay for the ink, toner, and paper for each of them, the rule would preclude a potential Pareto improvement. Setting the price equal to marginal social cost makes a lot more sense. Then no matter how many copies patrons wanted to make at that price, the outcome would be Pareto optimal.

- Thus when you know what the marginal social cost is going to be, as at the library, set a price. When you don't know what marginal social cost is go- ing to be but do know where it changes, as on the cruise ship, use a rule. For situations in between, use a rule for those more like the cruise ship case and a price or tax for those more like the library case.

- The second argument for using rules rather than Pigouvian taxes concentrates on enforcement costs—how hard it is to find out what people have done in order to tax or punish them appropriately. Community reporting is cheap and often important, and it's much easier for neighbors and the community in general to know whether someone is breaking a rule than whether she has paid appropriate taxes.

- For instance, consider Sunday liquor sales. The externalities these cause could be mitigated by either putting an extra tax on them or prohibiting them. If sales are prohibited, then it's easy for neighbors, clergy, and passersby to tell whether a liquor store is in compliance, and easy for a judge or jury to decide guilt or innocence. In contrast, a passerby on her way to church on a Sunday morning has no way to tell whether anyone is actually paying the appropriate Pigouvian surcharge on liquor.... Thus, so far as enforcement is concerned, rules are likely to be a cheaper and more effective way of dealing with Sunday liquor sales than are Pigouvian taxes.

- In general, then, when community monitoring is cheap and easy, and when tax collectors are likely to be lazy or corrupt, rules will often work better than taxes to enforce the desired behavior. Some other examples are hunting and fishing limits, rules on where buildings can be built, height limitations, restaurant and bar closing hours, noise ordinances, and laws on endangering the welfare of children.

2.2 Using taxes in the basic models of legal incentives

The market failures detailed in the basic models of Section 2 can all be eliminated through taxes rather than by legal rules. Basically, the Pigouvian approach of taxing externalities at their marginal external cost works provided the government has perfect enforcement power and perfect information about the payoffs. In the property model, the government can give efficient fencing incentives to the rancher simply by imposing a tax b for not building a fence (or a subsidy of b for building a fence). In the contract model, similarly, the government gives efficient incentives for product quality by setting a marginal subsidy of $b'(x)$ to the seller for quality (this is equivalent to a marginal tax of $b'(x)$ on lower quality). In tort law, a marginal tax of $p'(x)b$ as precaution decreases (a marginal subsidy of $p'(x)b$ as precaution increases) gives efficient incentives. In criminal law, finally, a 100% marginal tax on x is optimal.

For the four situations described in the basic legal incentives models, it's clear why taxes usually won't work. Even if their were funds available to administer these taxes, it would be difficult if not impossible to learn the payoffs. The rancher would disagree with the farmer's assessment of b in order to pay a lower tax. In the contract example, the government probably couldn't measure quality very effectively and wouldn't have good information about $b'(x)$. In tort, similarly, the government doesn't know $p(x)$ or b, especially when the harm doesn't occur. In criminal law, finally, the whole point is that the criminal cannot be taxed (punished) if not detected.

2.3 Police Powers

These are relevant excerpts from O'Flaherty, 2008, Chapter 8, Section II:

- Because regulations are often a good idea, it's not bad to have entities that can promulgate them. Because circumstances vary and change rapidly, and regulations need to adapt, the promulgating entities should have some discretion. Having someone around who has police powers can make all of us who are subject to those powers better off.

- Discretion and the power to coerce, however, are a dangerous combination. An entity with discretion and power can coerce all the rent for an entire city into its own hands simply by extortion . . . If there are some investments that people will make only if they can be rewarded for them later, then those investments will never be made. The coercive entity could never convince people that anything would ever restrain its capacity enough to let them harvest those rewards. Instead of making us all better off, police powers, if untrammeled, could make us all worse off. . . . Police powers, therefore, are circumscribed in many different ways.

- Procedural due process requires that governments follow some sort of already prescribed tests before they exercise police powers against someone.

- Substantive due process requires that the rules be reasonably related to the purpose of police powers—to promote the health, safety, morals, and general welfare of the community.

- Equal protection says that rules must be written and applied impersonally.

- The First Amendment protections of religion, speech, assembly, and po- litical activity also circumscribe some police powers.

- The final constitutional check on police powers is known as the takings clause. . . . This refers to an ancient practice called eminent domain. When a government is building a road or a firehouse, it can force landowners to sell it the property it needs—but it must pay them a fair price.

3 Bargaining

This section presents the basics of bargaining theory that will come in handy for this course.

3.1 Nash Bargaining Solution

Bargaining problems represent situations in which:

- There is a conflict of interest about agreements.
- Individuals have the possibility of concluding a mutually beneficial agreement.
- No agreement may be imposed on any individual without his approval.

There are two players – parties to a negotiation. Suppose the players $i = A, B$ have preferences represented by a utility function $u_i(x_i)$ where x_i is the outcome of the bargain for player i. We can think of the outcomes as payments x_i out of a fixed pie of size π, so $x_A + x_B \leq \pi$, If no agreement is reached, the players receive d_A and d_B, respectively. This is called the disagreement outcome or "threat point."

A pair of payoffs (u_A^*, u_B^*) is a Nash bargaining solution if it solves the following problem:

$$\max_{u_A, u_B} (u_A - d_A)(u_B - d_B)$$

subject to the constraints:

$$\begin{aligned} x_A + x_B &\leq \pi \\ u_A &\geq d_A \\ u_B &\geq d_B \end{aligned}$$

That is, it maximizes the product of the utility levels net of the disagreement utilities.

Example 1. Suppose there is \$100 on the table to be split between two players ($\pi = 100$). Players receive linear utility: $u(x_i) = x_i$. If no agreement is reached, players do not receive anything: $d_A = d_B = 0$. The feasibility constaint is $x_1 + x_2 \leq 100$, while the participation constraints are $x_A \geq 0$, $x_B \geq 0$. So the Nash bargaining problem is

$$\max_{u_A, u_B} u_A u_B$$

And we substitute $u_i = x_i$ into the constraints:

$$\begin{aligned} u_A + u_B &\leq 100 \\ u_A &\geq 0 \\ u_B &\geq 0 \end{aligned}$$

The constraint binds at the optimum, so you can substitute $u_B = 100 - u_A$. Maximizing $u_A(100 - u_A)$ gives $u_A^* = u_B^* = 50$.

Example 2. Same as Example 1, except $d_A = 20$. The Nash bargaining problem is

$$\max_{u_A, u_B} (u_A - 20)u_B$$

subject to

$$\begin{aligned} u_A + u_B &\leq 100 \\ u_A &\geq 20 \\ u_B &\geq 0 \end{aligned}$$

As before, you can substitute $u_B = 100 - u_A$. Maximizing $(u_A - 20)(100 - u_A)$ gives $u_A^* = 60, u_B^* = 40$.

Example 3. Same as Example 1, except $u_B = \sqrt{x_B}$. We account for the change in utility function is to change the substitution for $x_i$in the constraints. In particular, instead of $x_B = u_B$ we now have $x_B = u_B^2$. The Nash bargaining problem is

$$\max_{u_A, u_B} u_A u_B$$

subject to

$$\begin{aligned} u_A + u_B^2 &\leq 100 \\ u_A &\geq 0 \\ u_B^2 &\geq 0 \end{aligned}$$

The constraint binds, so substitute $u_A = 100 - u_B^2$ and differentiate

$$\begin{aligned}
(100 - u_B^2)u_B & \\
100u_B - u_B^3 & \\
100 - 3u_B^2 &= 0 \\
u_B^2 &= \frac{100}{3} \\
u_B^* &= \sqrt{\frac{100}{3}}
\end{aligned}$$

To get, u_A, plug back into the feasibility constraint to get

$$u_A = 100 - (\sqrt{\frac{100}{3}})^2 = \frac{200}{3}$$

The risk averse player gets a lower utility.

The Nash bargaining solution has two noteworthy features:

- *Symmetry*. If all agents are identical, then the gains are split equally.
- *Pareto efficiency*. There exists no feasible outcome in which one player is better off and the other player is at least as well off.

Here are two alternative solution methods:

- *Egalitarian solution*. The gains from bargaining are split equally among the agents regardless of the threat point.
- *Utilitarian solution*. The total utility is maximized:

$$\max_{u_A, u_B} u_A + u_B$$

These methods don't account for the threat point. That partly explains why they're less realistic in explaining bargaining behavior.

3.2 The alternative-offers model of bargaining

Consider two agents A and B who have to decide how to split some surplus π.[1] In law and economics, this could be the surplus from a contracted investment, for example, or the terms of a

[1] This discussion of alternating-offers bargaining is derived from Slantchev 2008, available at http://slantchev.ucsd.edu/courses/gt/05-extensive-form.pdf starting at pg. 38.

trial settlement. The rules of the game are that the agents take turns proposing a division of the pie. Begin with player A as the proposer; he offers $x \in [0, \pi]$ as the share he keeps. If B accepts the offer, then an agreement is made; A receives x and B receives $\pi - x$. If B rejects, the roles flip. B becomes the proposer and makes a counter-offer $y \in [0, \pi]$; if A accepts, B receives y and A receives $\pi - y$. And so on.

Players discount the future with a common discount factor $\delta \in [0, 1]$. While players disagree, neither receives anything (which means that if they perpetually disagree then each player's payoff is zero). If some player agrees on a partition$(x, \pi - x)$ at some time t, player A's payoff is $\delta^t x$ and player B's payoff is $\delta^t(\pi - x)$. This formalizes the idea that it is better to obtain an agreement sooner rather than later, due to impatience or due to the costs of bargaining/negotiations.

The subgame perfect Nash equilibrium is a set of equilibrium offers x^* and y^* that satisfies two conditions: (1) an equilibrium offer is always accepted, and (2) each player always makes the same offer. Consider some arbitrary period t when A is making an offer x^* to B. If B rejects, he must offer y^* due to (2), and A will accept it, due to (1). Therefore the payoff to B for rejecting an offer in equilibrium is δy^* (it is discounted by δ because he has to wait until the next period). Therefore B will reject any offer $\pi - x < \delta y^*$ and accept any offer $\pi - x > \delta y^*$. Further, assume that players accept when indifferent, so B will accept when $\pi - x = \delta y^*$. So Condition (1) requires that $\pi - x^* \geq \delta y^*$. But $\pi - x^* > \delta y^*$ would not be a best response for A because he could offer $\pi - x = \delta y^*$ and earn a higher payoff with B still accepting. In equilibrium, B is indifferent between accepting and rejecting:

$$\pi - x^* = \delta y^*$$

and symmetrically, A must be indifferent between accepting and rejecting:

$$\pi - y^* = \delta x^*$$

You can use substitution with these equations to solve for x^* and y^*:

$$\begin{aligned}
\pi - x^* &= \delta(\pi - \delta x^*) \\
\pi - \delta\pi &= x^* - \delta^2 x^* \\
(1-\delta)\pi &= (1-\delta^2)x^* \\
x^* &= \frac{(1-\delta)\pi}{1-\delta^2} \\
x^* &= \frac{(1-\delta)\pi}{(1+\delta)(1-\delta)} \\
x^* &= \frac{\pi}{1+\delta} = y^*
\end{aligned}$$

So the first mover (let's say A) receives

$$u_A = \frac{\pi}{1+\delta}$$

and the second mover (B) receives

$$u_B = \pi - \frac{\pi}{1+\delta} = \frac{\delta\pi}{1+\delta}$$

and the game ends at $t = 0$.

Note that for $\delta = 1$ the SPNE involves an even split ($x = \pi/2$).

3.3 Bargaining and Property Law

Let's return to the basic model of property law in Subsection 2.1. Consider the slightly revised payoff functions for the rancher A and the famer B, where w_A, w_B are the payment to each party and $x \in \{0, 1\}$ is whether or not the fence is built.

$$u_A = \omega_A - ax + w_A$$

$$u_B = \omega_B + bx + w_B.$$

Remember from above that if $a > b$ or under Regime 2 bargaining wasn't necessary. So assume Regime 1 and that $b > a$. The size of the pie is $b-a$, so the feasibility constraint is $w_A+w_B = b-a$. The disagreement outcomes, where $x = 0$, are $d_A = \omega_A$ and $d_B = \omega_B$.

3.3.1 Nash Bargaining

The Nash bargaining problem is

$$\max_{u_A,u_B} (u_A - d_A)(u_B - d_B)$$

The participation constraint for A is

$$\omega_A + w_A \quad \geq \quad \omega_A$$

The participation constraint for B is

$$\omega_B + w_B \quad \geq \quad \omega_B - b$$

We can rewrite the Nash bargaining problem in terms of w_A and w_B:

$$\max_{w_A,w_B}(\omega_A + w_A - \omega_A)(\omega_B + w_B - \omega_B$$

simplify:

$$\max_{w_A,w_B}(w_A)(w_B)$$

subject to

$$w_A + w_B = b - a$$

Substitute:

$$(b - a - w_B)(w_B)$$

Expand:

$$bw_B - aw_B - w_B^2$$

Differentiate:

$$\begin{aligned} b - a - 2w_B &= 0 \\ w_B &= \frac{b-a}{2} \end{aligned}$$

Which means

$$w_A = \frac{b-a}{2}$$

That is, an even split of the surplus.

3.3.2 Alternating Offers

Now take the alternating-offers approach. B proposes a contract (x, w). The "pie" to be divided is $\pi = b - a$. B proposes

$$w^* = a + \frac{\delta(b-a)}{1+\delta}$$

which A accepts, from the argument in 3.1, and builds the fence. The payoffs are

$$\begin{aligned} u_A &= \omega_A + (a + \frac{\delta(b-a)}{1+\delta}) - a \\ &= \omega_A + \frac{\delta(b-a)}{1+\delta} \end{aligned}$$

and

$$\begin{aligned} u_B &= \omega_B - (a + \frac{\delta(b-a)}{1+\delta}) + b \\ &= \omega_B + \frac{b-a}{1+\delta} \end{aligned}$$

If there are zero bargaining costs ($\delta = 1$), there is an even split of the surplus from building the fence: $w = a + \frac{b-a}{2}$. For positive bargaining costs ($\delta < 1$), B gets a first-mover advantage and gets more than half the surplus. The social optimum is still chosen in equilibrium (the fence is built only when $b > a$); adding this type of bargaining costs only affects the division of the surplus through the first-mover advantage.

3.4 Bargaining and Contract Law

Now let's look at our model of contracting from Subsection 2.2 in light of bargaining. As stated, the transaction is mutually beneficial for (x, w) such that $w \geq ax$ and $b(x) \geq w$, which can be written as $ax \leq w \leq b(x)$.

3.4.1 Alternating Offers

The interesting implication of adding a bargaining step to contracting is that it affects the division of the surplus. The party to make the first offer gets a better deal. In particular, the surplus from contracting in this model is is $\pi = b(x^*) - ax^*$. Applying the alternating-offers paradigm, if buyer B moves first he proposes

$$w^* = ax^* + \frac{\delta(b(x^*) - ax^*)}{1+\delta}$$

and the seller accepts. With perfect enforcement of contracts, seller chooses quality level $x = x^*$ and the payoffs are

$$u_A = \frac{\delta(b(x^*) - ax^*)}{1+\delta}$$

$$u_B = \frac{b(x^*) - ax^*}{1+\delta}$$

The buyer gets a higher payoff for $\delta < 1$. Conversely, if the seller A makes the offer, he gets a larger share of the surplus. What are the new participation constraints?

3.4.2 Nash Bargaining

As a final example on Nash bargaining, let's assume that the buyer's utility function is $u_B = b(x - w)$ and that the disagreement outcomes are $d_A > 0, d_B > 0$ rather than zero.

The Nash bargaining solution solves:

$$\max_w (w - ax - d_A)(b(x - w) - d_B)$$

Multiplying out this expression gives

$$wb(x - w) - axb(x - w) - d_A b(x - w) - wd_B + axd_B - d_A d_B$$

And taking first-order conditions for w:

$$
\begin{aligned}
b(x-w) - wb'(x-w) + axb'(x-w) + d_a b'(x-w) - d_B &= 0 \\
b(x-w) - d_B - (w - ax - d_A)b'(x-w) &= 0 \\
b(x-w) - d_B &= (w - ax - d_A)b'(x-w)
\end{aligned}
$$

which characterizes the Nash bargaining solution for w.

4 Property Law

4.1 Coase Theorem

O'Flaherty 2005, Chapter 7 Section V:

- The Coase theorem means that, in arguing for a government intervention, you have to show not only that what someone does affects someone else, but also that the parties cannot negotiate themselves out of the problem."
- The Coase theorem also suggests another type of policy, aside from taxes, subsidies, or tolls, to alleviate the problems caused by externalities—helping the parties negotiate.
- Lawyers sometimes refer to property as a "bundle of sticks." By that they mean that ownership is really a collection of rights. What we call "owning a tire iron" consists of the right to keep other people from using it, the right to carry it in your trunk, the right to change your own tires with it, ... and so on. These are the sticks that are in the bundle. "Owning a tire iron," in most of the United States today, does not include the right to use it to remove other people's tires unless they say you can, ... the right to throw it at imaginary rabbits on a crowded sidewalk, ... and so on. These are some of the sticks not in the bundle labeled "owning a tire iron."
- The Coase theorem says that, so far as efficiency is concerned, if people can negotiate, it doesn't matter which sticks are in which bundle. But every stick has to be assigned unambiguously to one and only one bundle, and it's the government's job to assign them.
- In more technologically dynamic arenas like the Internet, organ transplants, and the deep seabed, sticks that are in no bundle are constantly turning up. Efficient negotiation can take place only after those sticks have found a home in some bundle.

4.2 Poorly Defined Property Rights

Recall the basical model of property rights with a landowner A and his neighbor, B. A chooses a binary action, $x \in \{0, 1\}$. A and B begin with endowments ω_A and ω_B. A monetary transfer between the parties is represented by w. The cost of x to A is parametrized by a and the benefit of x to B is parametrized by b. The payoff functions for A and B are

$$u_A = \omega_A + w - ax$$

$$u_B = \omega_B - w + bx.$$

Previously we looked at the cases of Regime 1 (no fence building requirement) and Regime 2 (fences required but exemptions allowed). We showed that the outcome didn't matter when negotiation was allowed.

Now consider the case where the property rights are not well-defined. There is a propability p that a court will follow Regime 1, and a probability $1-p$ that a court will follow Regime 2. In Regime 2, the Farmer doesn't want an agreement; in Regime 1, he does.

4.3 Transaction Costs

Recall the basical model of property rights with a landowner A and his neighbor, B. A chooses a binary action, $x \in \{0, 1\}$. A and B begin with endowments ω_A and ω_B. A monetary transfer between the parties is represented by w. The cost of x to A is parametrized by a and the benefit of x to B is parametrized by b.

Now extend the model to allow for a cost $\gamma \geq 0$ of writing and enforcing an agreement about the fence. When there is an agreement, both parties have to pay γ; without an agreement they don't pay the cose. The payoff functions for A and B without bargaining are

$$u_A = \omega_A + w - ax$$

$$u_B = \omega_B - w + bx$$

and with bargaining are

$$u_A = \omega_A + w - ax - \gamma$$

$$u_B = \omega_B - w + bx - \gamma.$$

Recall that the case where we have bargaining is Regime 1, where $b > a$. Now we require that $b - 2\gamma > a$ in order for a bargain to occur. For $b < a$, no bargain occurs and that's efficient. For $b > a + 2\gamma$, the transaction occurs and its an efficient outcome. For $b \in (a, a + 2\gamma)$, no bargain occurs due to transaction costs, but the outcome is inefficient. In that range of b, moving to Regime 2 improves efficiency.

5 Tort Law

5.1 Bilateral Precaution Model

As discussed in the slides, the various doctrines of tort liability put more or less incentive pressure on the injurer and victim. Here we will give a formal model of tort liability in which both the plaintiff (victim) and the defendant (injurer) can set precaution levels.

The parties are the defendant A and the plaintiff B. The defendant chooses precaution x and the plaintiff chooses precaution y. The probability of the accident is given by

$$p(x, y) = p(x) + q(y)$$

The harm from the accident is given by H. Here we use cost functions rather than utility functions:

$$\begin{aligned} c_A &= x \\ c_B &= \begin{cases} y & \text{no accident} \\ y - H & \text{accident} \end{cases} \end{aligned}$$

To minimize the social cost of the accident, one would minimize the expected social cost

$$\begin{aligned} SC = \mathbb{E}(c_A + c_B) &= x + y + p(x, y)H \\ &= x + y + [p(x) + q(y)]H \end{aligned}$$

The socially optimal level x^* of the injurer's precaution is obtained by the derivative for x:

$$1 = -p'(x^*)H$$

and similarly for y^*:

$$1 = -q'(y^*)H$$

Suppose that the injurer has to pay damages D in the event of an accident. The expected cost functions are

$$\mathbb{E}(c_A|x) = x + [p(x) + q(y)]D$$

and

$$\mathbb{E}(c_B|y) = y + [p(x) + q(y)](H - D)$$

The case of $D = 0$ is the case of no liability. The injurer solves

$$\min_{x \geq 0} x$$

and chooses $x = 0$. The victim solves

$$\min_{y \geq 0} y + q(y)H$$

chooses the social optimum $y = y^*$ such that $1 = -p'(y^*)H$. The no liability regime is good for victim's incentives but not for injurer's incentives.

The case of $D = H$ is the case of strict liability. The injurer solves

$$\min_{x \geq 0} x + p(x)H$$

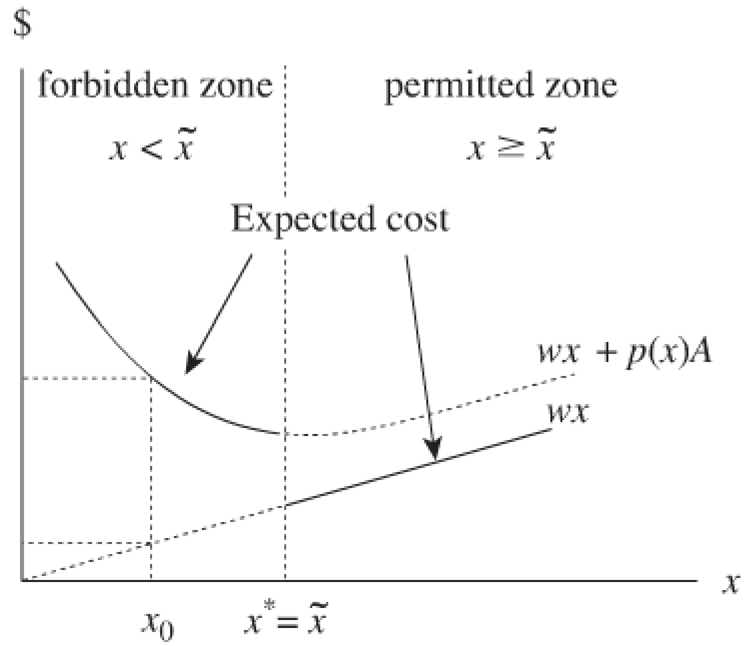
and chooses the social optimum $x = x^*$. The victim solves

$$\min_{y \geq 0} y$$

and chooses $y = 0$. Strict liability is good for the injurer's incentive but bad for the victim's incentive.

Now consider a simple negligence rule. The injurer is liable if $x < \tilde{x}$, where $\tilde{x}$ is the negligence standard – the minimum precaution level specified by tort law below which the injurer will be held negligent in the event the harm occurs. Ideally, $\tilde{x} = x^*$, which we'll assume for now. The injurer's expected cost is now

$$\mathbb{E}(c_A|x) = \begin{cases} x + [p(x) + q(y)]H & \text{if } x < x^* \\ x & \text{if } x \geq x^* \end{cases}$$



The two possible outcomes are $x = 0$ or $x = x^*$. If $x = 0$, the victim is insured and chooses $y = 0$. The expected utilities are

$$\mathbb{E}(c_A|x = 0) = [p(0) + q(0)]H$$

and

$$\mathbb{E}(c_A|x = x^*) = x^*$$

If $[p(0) + q(y^*)]H > x^*$, the injurer chooses the social optimum. Given $x = x^*$, the victim faces the full costs of the injury:

$$\mathbb{E}(c_B) = y + [p(x) + q(y)]H$$

and will choose $y = y^*$.

Next consider contributory negligence. The injurer will be treated as negligent and held responsible for the harm if he chooses $x < \tilde{x}$, but not if $y < \tilde{y}$, where $\tilde{y}$ is the precaution standard for the plaintiff. If $\tilde{y} \leq y^*$, nothing changes relative to the negligence case because given the assumptions on the production function (that is, perfect substitutability in precaution), the plaintiff chooses y^*anyway.

We can model comparative negligence in the following intuitive way. The injurer is liable for the full cost of the harm H if $x < \tilde{x}$ and $y \geq \tilde{y}$, but proportionally liable if $x < \tilde{x}$ and $y < \tilde{y}$. An intuitive assignment rule would have the injurer pay

$$D(x, y) = \frac{\tilde{x} - x}{\tilde{x} - x + \tilde{y} - y}$$

which is basically the ratio of negligence levels between the parties. If we have $\tilde{x} = x^*$, $\tilde{y} = y^*$, the victim's cost function given $x = x^*$ is

$$\mathbb{E}(c_A) = y + (p(x^*) + q(y))H$$

so he chooses $y = y^*$. Similarly, the injurer's expected cost given $y = y^*$ is

$$\mathbb{E}(c_A|x) = \begin{cases} x + [p(x) + q(y)]H & \text{if } x < x^* \\ x & \text{if } x \geq x^* \end{cases}$$

and he will choose $x = x^*$. So comparative negligence also has the same outcome as negligence under these assumptions.

This is the key point: given perfect compensation and a legal standard equal to the efficient levl of care, every form of negligence rule gives the injurer and victim incentives for efficient precaution.

5.2 Separable Precaution

Assume $p(x) = \frac{1}{\alpha}(1-x)^\alpha$ and $q(y) = \frac{1}{\beta}(1-y)^\beta$ where $\alpha > 1$,$\beta > 1$. Find the optimal negligence standards $\tilde{x}$ and $\tilde{y}$. Find the chosen precaution levels, probability of accident, and the expected utilities under no liability, strict liability, negligence, contributory negligence, and comparative negligence.

Solution.

$$\begin{aligned} SC = \mathbb{E}(c_A + c_B) &= x + y + p(x,y)H \\ &= x + y + [\frac{1}{\alpha}(\frac{1}{2} - x)^\alpha + \frac{1}{\beta}(\frac{1}{2} - y)^\beta]H \end{aligned}$$

The socially optimal precautions are obtained by

$$\begin{aligned} 0 &= 1 - (\frac{1}{2} - x)^{\alpha - 1}H \\ (\frac{1}{H})^{\frac{1}{\alpha-1}} &= \frac{1}{2} - x \\ x^* &= \frac{1}{2} - H^{\frac{1}{1-\alpha}} \end{aligned}$$

and

$$\begin{aligned}
0 &= 1-(\frac{1}{2}-y)^{\beta-1}H \\
(\frac{1}{H})^{\frac{1}{\beta-1}} &= \frac{1}{2}-y \\
y^* &= \frac{1}{2}-H^{\frac{1}{1-\beta}}
\end{aligned}$$

Note that both x^* and y^* are increasing with H because $\alpha > 1$, $\beta > 1$.

Under no liability, we have

$$\begin{aligned}
x_{NL} &= 0 \\
y_{NL} &= y^*
\end{aligned}$$

Under strict liability:

$$\begin{aligned}
x_{NL} &= x^* \\
y_{NL} &= 0
\end{aligned}$$

Under negligence, two possible injurer choices are $x = 0$ or $x = x^*$. If $x = 0$, the victim is insured and chooses $y = 0$.

The expected utilities are

$$\begin{aligned}
\mathbb{E}(c_A|x=0) &= [p(0)+q(0)]H \\
&= [\frac{1}{\alpha}(\frac{1}{2})^{\alpha}+\frac{1}{\beta}(\frac{1}{2})^{\beta}]H \\
&= \frac{H}{\alpha 2^{\alpha}}+\frac{H}{\beta 2^{\beta}}
\end{aligned}$$

and

$$\mathbb{E}(c_A|x=x^*) = \frac{1}{2}-H^{\frac{1}{1-\alpha}}$$

The condition on the harm level for the injurer to choose x^* is

$$\frac{H}{\alpha 2^{\alpha}}+\frac{H}{\beta 2^{\beta}} > \frac{1}{2}-H^{\frac{1}{1-\alpha}}$$

which we will assume holds. the injurer chooses the social optimum. Given $x = x^*$,

5.3 Substitutable Precaution

The probability of harm H is $\frac{1}{\beta}(1-x-y)^{\beta}$ and the costs of precaution are 1 and α, respectively.

$$x + \alpha y + \frac{1}{\beta}(1-x-y)^{\beta} H$$

so we have

$$0 = 1 - (1-x-y)^{\beta-1} H$$

and

$$0 = \alpha - (1-x-y)^{\beta-1} H$$

If $\alpha < 1$, victim has cheaper precaution and victim should take precaution; if $\alpha > 1$, victim has more expensive precaution and injurer should take precaution. Say $y = 0$, then

$$\begin{aligned} 0 &= 1 - (1-x)^{\beta-1} H \\ (1-x)^{\beta-1} H &= 1 \\ 1 - x &= H^{-\frac{1}{\beta-1}} \\ x^* &= 1 - H^{-\frac{1}{\beta-1}} \end{aligned}$$

Similarly if $x = 0$ then

$$y^* = \alpha - H^{-\frac{1}{\beta-1}}$$

With costless bargaining between the parties, they will figure out α and write a contract specifying who should take precaution. But otherwise, both parties would want to be the one taking no precaution.

No liability results in $x = 0$, $y = y^*$. This is optimal if $\alpha < 1$.

Strict liability results in $x = x^*$, $y^* = 0$. This is optimal if $\alpha > 1$.

Negligence with $\tilde{x} = x^*$ results in $x = x^*$. Then expected costs for y is

$$\begin{aligned} \alpha y + \frac{1}{\beta}(1-x-y)^{\beta} H &= \alpha y + \frac{1}{\beta}(1 - (1 - H^{-\frac{1}{\beta-1}}) - y)^{\beta} H \\ &= \alpha y + \frac{1}{\beta}(H^{-\frac{1}{\beta-1}} - y)^{\beta} H \end{aligned}$$

with FOC

$$\begin{aligned}
0 &= \alpha - (H^{-\frac{1}{\beta-1}} - y)^{\beta-1} H \\
(H^{-\frac{1}{\beta-1}} - y)^{\beta-1} &= \frac{\alpha}{H} \\
H^{-\frac{1}{\beta-1}} - y &= (\frac{\alpha}{H})^{\frac{1}{\beta-1}} \\
y &= H^{-\frac{1}{\beta-1}} - (\frac{\alpha}{H})^{\frac{1}{\beta-1}}
\end{aligned}$$

which could be greater than zero, in which case the precaution levels would be higher than the social optimum.

Contributory negligence with $\tilde{y} = y^*$ results, symmetrically, with

$$x = H^{-\frac{1}{\beta-1}} - (\frac{1}{H})^{\frac{1}{\beta-1}}$$

Contributory negligence is preferred to simple negligence when $\alpha < 1$.

5.4 Court Errors

The parties are the defendant A and the plaintiff B. The defendant chooses precaution x and the plaintiff chooses precaution y. The probability of the accident is given by

$$p(x, y) = p(x) + q(y)$$

The harm from the accident is given by H. Here we use cost functions rather than utility functions:

$$\begin{aligned}
c_A &= x \\
c_B &= \begin{cases} y & \text{no accident} \\ y - H & \text{accident} \end{cases}
\end{aligned}$$

In the event of a lawsuit, the court makes the correct decision with probability $\alpha < 1$.

With a negligence standard, the court finds the negligence standard to be $x^* + \epsilon$, where ϵ is uniformly distributed between $[-\beta, \beta]$.

5.5 Vicarious liability 1

The players are an employee E, an employer P, and a victim V. The employee chooses precaution x, the employer chooses precaution y. The employee has assets A. The probability of the accident is

$$p(x, y)$$

The administrative costs of filing suit against a party is given by C. Let the number of people sued be given by n. The social costs are

$$SC = x + y + p(x, y)H + nC$$

The social optimum is

$$\begin{aligned} -1 &= p_x H \\ -1 &= p_y H \\ n &= 0 \end{aligned}$$

Under no liability, or if not sued, the employee and employer choose $x = 0$ and $y = 0$. Social costs are $p(0, 0)H$. Under strict liability for the employee, we have

Example 1. Let $H = 10$, and

$$p(x, y) = \frac{1}{2}(1 - x)^2 + \frac{1}{2}(1 - \alpha y)^2$$

The social costs are

$$SC = x + y + [\frac{1}{2}(1 - x)^2 + \frac{\alpha}{2}(1 - y)^2]10 + nC$$

Optimality means

$$\begin{aligned} 1 &= 10(1 - x) \\ \frac{1}{10} &= 1 - x \\ x^* &= \frac{9}{10} \end{aligned}$$

$$\begin{aligned} 1 &= 10\alpha(1 - y) \\ \frac{1}{10\alpha} &= 1 - y \\ y^* &= 1 - \frac{1}{10\alpha} \end{aligned}$$

and $n^* = 0$.

Without liability, we have $x = 0$, $y = 0$, $n = 0$.

With strict liability and lawsuits against both parties, we have x^* and y^*.

5.6 Vicarious liability 2

The players are an employee, employer, and victim. The employee chooses precaution x and has assets A. The principal can monitor the employer's precaution and offer a wage $w + bx$, where $b \geq 0$ is the payment for precaution. The probability of the accident is $p(x)$. The social costs are

$$SC = x + p(x)H$$

The social optimum is

$$-1 = p_x H$$

Under no liability, the employee chooses $x = 0$ and the employer chooses $b = 0$. Social costs are $p(0, 0)H$. Under strict liability for the employee, we have

Example 1. Let $H = 10$ and $p(x) = \frac{1}{1+x}$. The social optimum is

$$\begin{aligned} 0 &= 1 - (1 + x)^{-2}4 \\ \frac{1}{4} &= (1 + x)^{-2} \\ 2 &= 1 + x \\ x &= 1 \end{aligned}$$

without liability we have $x = 0$.

With strict liability we have employee costs

$$(1 - b)x + A(1 + x)^{-1}$$

which is solved as

$$\begin{aligned} 1 - b &= A(1 + x)^{-2} \\ (\frac{A}{1 - b})^{\frac{1}{2}} &= 1 + x \\ x &= (\frac{A}{1 - b})^{\frac{1}{2}} - 1 \end{aligned}$$

Without vicarious liability, the employer will set $b = 0$ and we get $x = \sqrt{A} - 1$.

We can reach the social optimum by setting b such that

$$\begin{aligned}
1 &= (\frac{A}{1-b})^{\frac{1}{2}} - 1 \\
2 &= (\frac{A}{1-b})^{\frac{1}{2}} \\
4 &= \frac{A}{1-b} \\
1-b &= \frac{A}{4} \\
b &= 1 - \frac{A}{4}
\end{aligned}$$

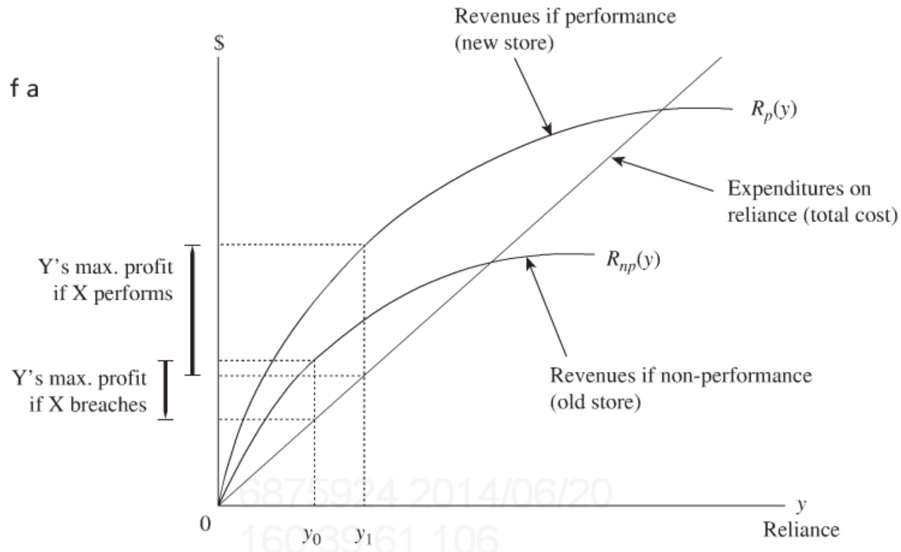
The lower the employee's assets, the higher the wage needed to induce compliance. If $A = 4$, no need for vicarious liability.

6 Contract Law

6.1 Efficient breach and efficient reliance example

These are notes based on Cooter and Ulen, Chapter 9 Appendix.

- Xavier promises to perform a service for Yvonne, for example expanding her store. He spends x on performing, and the probability of performance is $p(x)$.
- Yvonne relies on the contract, for example by expanding inventory. The reliance expenditures are given by y. Yvonne's payoff goes up with reliance and is complementary with whether performance occurs:



- Efficiency requires choosing x and y to maximize Yvonne's expected profits minus Xavier's expenditures. Formally:

$$\max_{x,y} p(x)F(y) + (1 - p(x))G(y) - x - y$$

FOC for x is

$$1 = p_x[F(y) - G(y)]$$

FOC for y is

$$1 = pF_y + (1 - p)G_y$$

These equations determine the optimal x^* and y^*

- Expectations damages D_E puts Yvonne in the same position as if Xavier performed, that is profits with performance minus profits without performance:

$$\begin{aligned} D_E &= (F(y) - y) - (G(y) - y) \\ &= F(y) - G(y) \end{aligned}$$

- Reliance damages D_R puts Yvonne in the same position as if she had not signed a contract. Assume that without contract her reliance expenditures would have been y_0 and her profits would have been $G(y_0) - y_0$. So her reliance damages for reliance level y would be

$$D_R = (G(y_0) - y_0) - (G(y) - y)$$

  and we know that $D_R < D_E$

- We could also compute opportunity-cost damages if we specified the terms of a next-best contract.

- Given damages D, Xavier solves

$$\min_x x + (1 - p(x))D$$

  which gives

$$1 = p_x D$$

  Under expectations damages, we have

$$1 = p_x[F(y) - G(y)]$$

  Under reliance damages we have

$$1 = p_x[G(y_0) - y_0) - (G(y) - y)]$$

  Expectations damages is optimal: $x_E = x^*$; because $D_R < D_E$, reliance damages give inefficiently low effort incentives: $x_R < x^*$.

- Given damages D, Yvonne solves

$$\max_y pF(y) + (1 - p)[G(y) + D(y)] - y$$

- Under perfect expectations damages (which uses y^*), Yvonne maximizes

$$pF(y) + (1-p)[G(y) + F(y^*) - G(y^*)] - y$$

which results in privately optimal reliance

$$1 = pF_y + (1-p)G_y$$

which is efficent.

- Under imperfect expectations damages (which uses y), Yvonne maximizes

$$\begin{aligned} pF(y) + (1-p)[G(y) + F(y) - G(y)] - y &= pF(y) + (1-p)[F(y)] - y \\ &= F(y) - y \end{aligned}$$

which results in reliance

$$1 = F_y$$

Which is suboptimally high

- Under reliance damages, Yvonne maximizes

$$\begin{aligned} pF(y) + (1-p)[G(y) + (G(y_0) - y_0) - (G(y) - y)] - y &= pF(y) + (1-p)[G(y_0) - y_0 + y)] - y \\ &= pF(y) + (1-p)[G(y_0) - y_0] - py \end{aligned}$$

which results in reliance

$$\frac{1}{p} = F_y$$

which is even higher than imperfect damages.

- Finally, under no damages (or restitution), we would have Yvonne maximizing

$$pF(y) + (1-p)[G(y)] - y$$

which also results in privately optimal reliance

$$1 = pF_y + (1-p)G_y$$

6.2 Another model

In contract theory, A can be thought of as the seller, and B the buyer.

With externalities, introduce C as a third party.

Consider a seller A and a buyer B. A promises to sell to B a good with quality q at price p, but B will observe the quality only after paying p. The cost to A of q is $c(q)$, which is increasing and convex. So the profit from the transaction for A is

$$\pi_A(p, q) = p - c(q).$$

The buyer's utility is given by

$$u_B(p, q) = u(q) - p$$

where $u(\cdot)$ is increasing and concave. We normalize the outside options for each agent to zero. In consequence, the transaction is mutually beneficial, and therefore both parties would agree to it, for any (p, q) such that $p \geq c(q)$ and $u(q) \geq p$, which can be written more succinctly as $c(q) \leq p \leq u(q)$. These are the participation constraints for buyer and seller.

At the social optimum, the marginal benefit of quality equals the marginal cost. The optimal quality level q^* satisfies

$$u'(q^*) = c'(q^*)$$

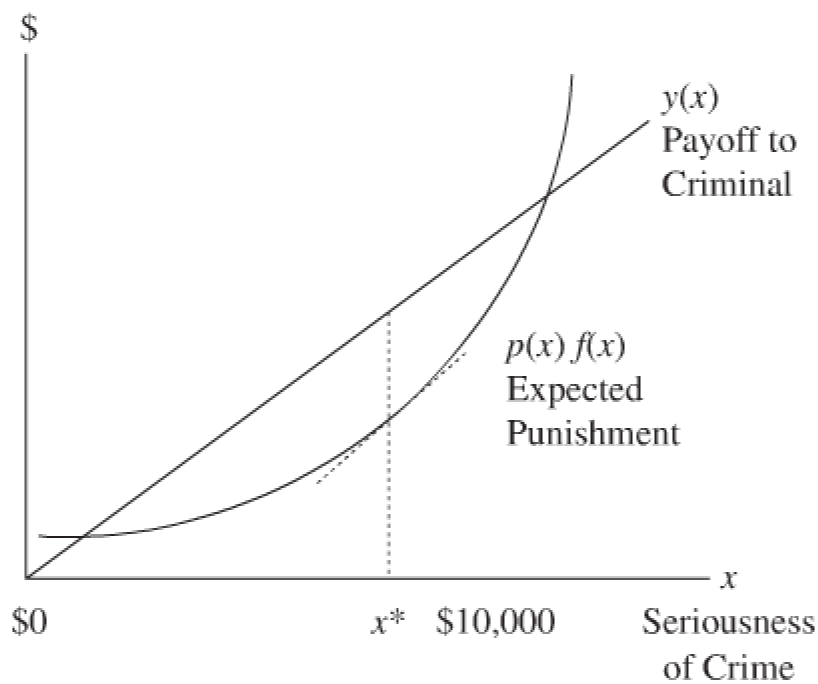
which, along with any p where $c(q*) \leq p \leq u(q^*)$, specifies the possible social optima.

7 Criminal Law

7.1 Model of options for reducing crime

In the intro model of crime, the government could affect crime rates only by increasing the probability of detection. Now we will consider two other policy options: 1) increasing the punishment for convicted criminals, and 2) subsidizing non-crime activity to increase the opportunity cost of crime.

The criminal chooses what proportion of time to spend on crime, $x \in [0, 1]$. So the amount of time spent on non-crime activities is $1 - x$. The benefit from crime is given by the function $b(x)$, which increases concavely with x. The benefit from non-crime is given by $(a + s)(1 - x)$, where a is a positive real number and s is the government subsidy to non-crime. The punishment for a crime level x is given by $f(x)$, which is assumed to be increasing in x. Finally, the probability of detection and punishment is given by $p(x)$, which is also increasing in the amount of time spent on crime. The expected sanction for crime level x is $p(x)f(x)$.



The criminal's expected payoff is

$$\mathbb{E}(u(x)) = b(x) + (a + s)(1 - x) - p(x)f(x).$$

The derivative for x gives the optimality condition

$$b_x = a + s + p_x f(x) + p(x) f_x$$

which equates the marginal benefit of crime with the marginal cost.

- Because $b(x)$ is strictly concave, $b'(x)$ is decreasing, so increasing s will decrease x.
- Similarly, increasing the probability of detection $p(x)$ will decrease x
- Similarly, increasing the sanction $f(x)$ will decrease x.

To see this more clearly, assume that y is the expenditure on police and z is the expenditure on prisons, where now we have $p(x, y)$ and $f(x, z)$. The criminal's new expected utility is

$$\mathbb{E}(u(x)) = b(x) + (a + s)(1 - x) - p(x, y) f(x, z).$$

With first-order condition

$$b_x = a + s + p_x(x, y) f(x, z) + p(x, y) f_x(x, z)$$

Example. Let $b(x) = \frac{1}{3}x^{\frac{1}{3}}$, $u(x) = \frac{a+s}{3}(1-x)^{\frac{1}{3}}$,$p(x, y) = \frac{3}{2}x^{\frac{2}{3}}y^{\frac{1}{2}}$, and $f(x, z) = x^{\frac{2}{3}}z^{\frac{1}{2}}$. We have

$$b_x = x^{-\frac{2}{3}}$$
$$u_x = -(a+s)(1-x)^{-\frac{2}{3}}$$

$$p_x = x^{-\frac{1}{3}}y^{\frac{1}{2}}$$
$$f_x = \frac{2}{3}x^{-\frac{1}{3}}z^{\frac{1}{2}}$$

The criminal's optimal crime is defined by

$$\begin{aligned}
x^{-\frac{2}{3}} &= (a+s)x^{-\frac{2}{3}} + (x^{-\frac{1}{3}}y^{\frac{1}{2}})(x^{\frac{2}{3}}z^{\frac{1}{2}}) + (\frac{3}{2}x^{\frac{2}{3}}y^{\frac{1}{2}})(\frac{2}{3}x^{-\frac{1}{3}}z^{\frac{1}{2}}) \\
x^{-\frac{2}{3}} &= (a+s)x^{-\frac{2}{3}} + x^{\frac{1}{3}}y^{\frac{1}{2}}z^{\frac{1}{2}} + x^{\frac{1}{3}}y^{\frac{1}{2}}z^{\frac{1}{2}} \\
(1-a-s)x^{-\frac{2}{3}} &= 2x^{\frac{1}{3}}y^{\frac{1}{2}}z^{\frac{1}{2}} \\
x^{-1} &= \frac{2y^{\frac{1}{2}}z^{\frac{1}{2}}}{(1-a-s)} \\
x &= \frac{(1-a-s)}{2y^{\frac{1}{2}}z^{\frac{1}{2}}}
\end{aligned}$$

8 Constitutional Law

This section discusses Maskin and Tirole (AER, 2004).

Under RD, the official chooses a with probability $p\pi + (1-p)(1-\pi)$. The posterior probability that the politician is congruent given the choice of a is

$$\frac{p\pi}{p\pi + (1-p)(1-\pi)}$$

This is also the expected utility from retaining the politician. The probability the official chooses b is $p(1-\pi) + (1-p)\pi$. The official is removed in this case and another official is elected, who is congruent with probability π and gives an expected utility of π. Therefore the expected welfare in the second period is

$$[p\pi + (1-p)(1-\pi)][\frac{p\pi}{p\pi + (1-p)(1-\pi)}] + [p(1-\pi) + (1-p)\pi][\pi]$$

which simplifies to

$$p\pi + [p(1-\pi) + (1-p)\pi]\pi$$

which we add to the first period payoff to get the expected utility for representative democracy:

$$\pi + p\pi + [p(1-\pi) + (1-p)\pi]\pi$$

which is strictly better than judicial power (greater than 2π). It is better than direct democracy when

$$\begin{aligned}
\pi + p\pi + [p(1-\pi) + (1-p)\pi]\pi &> 2p \\
\pi + p\pi + p\pi - p\pi^2 + \pi^2 - p\pi^2 &> 2p \\
\pi + \pi^2 &> p(2 - 2\pi + 2\pi^2) \\
p &< \frac{\pi(1+\pi)}{2(1-\pi+\pi^2)}
\end{aligned}$$