Lecture Notes on Law and Economics: Background

Elliott Ash

July 23, 2015

1 Math Background

1.1 Calculus

This subsection outlines the main results from calculus that we will use.

- Partial Differentiation (Product Rule, Chain Rule)
  - Find partial derivatives of $x^2 f(x)\sqrt{y}$
  - Find partial derivatives of $(xy - \sqrt{y})^2$
  - Find partial derivatives of $u(y - f(x))$
- For univariate functions, concavity means that the second derivative is negative, while convexity means that the second derivative is positive.
- Maximizers and minimizers are preserved for monotonic transformations of the objective function. For example, taking logs.
  - Maximize $x^\alpha y^\beta$ subject to $x + py \leq w$ by taking logs

1.2 Statistics

Consider a random variable X with expectation $\mathbb{E}(X)$. Then we have the following properties of scaling and addition. where a and b are real numbers:

$$\mathbb{E}(aX + b) = a\mathbb{E}(X) + b$$

You can't do this with non-linear functions. But we do have the following relationship between concavity and expectation.

- If a function $f(\cdot)$ is concave, $\mathbb{E}(f(x)) \leq f(\mathbb{E}(x))$ (with a strict inequality if $f(\cdot)$ is strictly concave). An agent with a concave utility function is risk-averse.

- If a function $f(\cdot)$ is convex, $\mathbb{E}(f(x)) \geq f(\mathbb{E}(x))$ (with a strict inequality if $f(\cdot)$ is strictly convex). An agent with a convex utility function is risk-loving.

A uniformly distributed random variable X with support on the interval (a, b) has $\mathbb{E}(X) = \frac{a+b}{2}$, which is the midpoint of the interval.

2 Economics background

This section gives the background in economics, notably in optimization, expected utility, game theory, and agency theory.

2.1 Elasticities

An "elasticity" refers to how one variable changes when another variable changes. It is useful because it does not have a unit of measurement, so you can sensibly measure things like the effect of temperature on crime levels.

2.2 Market failures

Be familiar with the standard market failure concepts: externalities, public goods, monopoly, etc., and the standard solutions in microeconomics.

2.3 Utility Maximization

Consider the following basic utility maximization problem. An individual has a utility or objective function given by

$$u(x_a, x_b)$$

and a budget constraint

$$p_a x_a + p_b x_b \leq y.$$

There are usually multiple ways to solve problems like this. Perhaps the easiest is to solve for x_b:

$$x_b = \frac{y - p_a x_a}{p_b}$$

and then plug into the original utility function:

$$u(x_a, \frac{y - p_a x_a}{p_b})$$

Then take the derivative and set equal to zero, which gives you x_a^*:

$$\frac{\partial u(x_a, \frac{y - p_a x_a}{p_b})}{\partial x_a} = 0$$

then plug x_a^* back into the budget constraint to get x_b^*:

$$x_b = \frac{y - p_a x_a^*}{p_b}$$

2.4 Expected Utility Theory

We use the standard expected utility model. Consider a gamble with payoff $y = x$ with probability p and $y = B$ with probability $1 - p$. The expected value of the gamble is $\mathbb{E}(y) = px + (1-p)B$. Now take an agent with utility function $u = \log(y) - x$, where y is the payout from the gamble. The expected utility function is

$$p[\log(x) - x] + (1-p)[\log(B) - x].$$

How much will this guy invest? He takes the first order condition to maximize expected utility:

$$\begin{aligned} \frac{p}{x} - p + 1 - p &= 0 \\ p &= x^* \end{aligned}$$

which gives the optimal gambling investment.

2.5 Pareto Efficiency

[see Cooter and Ulen Question 2.19]

2.6 Game Theory

- A Nash equilibrium is a set of strategies where no player could get a higher payoff from unilaterally changing his strategy.
- A subgame perfect Nash equilibrium is the extensive-form NE where players play best responses at all nodes.
  - In this course, we only care about SPNE's in extensive-form games
- Solve for the NE in this game if it's simultaneous move. Draw the extensive form for Player 1 moving first and find the SPNE.

		player 2	
		L	R
player 1	A	1, 2	1, 2
	B	0, 0	2, 1

2.7 Potential topics for later

- Statistics
  - Bayes Rule
  - Normally distributed random variable: $X \sim \mathcal{N}(\mu, \sigma^2)$, mean μ and variance σ^2. Formulas for truncated normals. Bayes rule for updating a normally distributed prior from normally distributed signals.
- Game Theory
  - Mixed strategies
  - Player Types (Incomplete Information)
  - Perfect Bayesian Equilibrium
- Principal-agent model
  - Incentive compatibility and participation constraints
  - Moral hazard
  - Adverse selection
  - Insurance